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THEORY OF PLASMA OSCILLATIONS WITH PAIR CORRELATIONS\*

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## ABSTRACT

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Effects of electron-ion and electron-electron correlations on plasma oscillations have been studied by obtaining an approximate solution for the first two members of the BBGKY hierarchy. Electron-ion correlations are shown to be especially important, for they produce, in the long wavelength limit: (1) a damping independent of wave number  $k$ ; (2) a small correction to the plasma wave frequency which is independent of wavelength and thus modifies the Langmuir plasma frequency. The damping which is wavelength independent predominates over Landau damping and is in close agreement with the damping obtained from high frequency conductivity calculations.<sup>4-7</sup>

For both the damping and the dispersion relation, electron-electron correlations are found to appear first in terms proportional to  $k^2$ . These terms, as well as the  $k^2$  terms from electron-ion correlations, have also been computed. Our results are compared with those made previously.<sup>1,2</sup>

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## I. Introduction

There have been several attempts recently<sup>1-3</sup> to examine the effects of pair correlations on plasma oscillations by using the BBGKY hierarchy. In all of these cases, however, only electron-electron correlations were considered and electron-ion correlations were ignored. It was found by all of these authors that, in the long wavelength limit, the largest damping was proportional to  $k^2$ , where  $k$  is the wave number. Also, it was found<sup>1,2</sup> that there is a small correction to the plasma wave frequency which is proportional to  $k^2$ . Both Ichikawa<sup>1</sup> and Willis<sup>2</sup> approximated the equation for the two electron correlation function by dropping numerous terms. Furthermore, although they solved the same equation, they obtained considerably different numerical results. Gorman and Montgomery<sup>3</sup> have obtained a more complete solution for the damping but have not yet presented numerical results.

We have also used the BBGKY hierarchy to study the effect of pair correlations on plasma oscillations. Although our solution of the binary correlation equations is approximate, our analysis is more complete than either Ichikawa or Willis, both by considering electron-ion correlations and by including in the electron-electron correlation equation many terms which they dropped. We have obtained, in the long wavelength limit:

1. A damping independent of wave number  $k$  resulting from electron-ion correlations;
2. damping proportional to  $k^2$  from both electron-electron and electron-ion correlations;

3. a small correction to the usual plasma frequency resulting from electron-ion correlations which is independent of wavelength;
4. corrections to the dispersion relation which are proportional to  $k^2$ .

A damping independent of the wave number has been found previously<sup>4-7</sup>. All of these authors obtained the high frequency conductivity from which, as shown by DuBois, Gilinsky and Kivelson<sup>6</sup>, the plasma wave damping can be determined. In references 4, 6 and 7, diagrammatic techniques were employed for the calculation, while in reference (5) the solution of the BBGKY hierarchy for a homogeneous plasma with an externally applied oscillating electric field was used. Thus far, only the conductivity  $\sigma(\omega)$  which is independent of wavelength has been given explicitly\*. For the infinite ion mass limit, the results of these authors are in agreement. The numerical result for the damping obtained from the conductivity<sup>4</sup> is in reasonably good agreement with the result which we have obtained. Further discussion of this matter will be given in Section IV.

In Section II, we present the basic mathematical formulation. The approximations used to solve the pair correlation equations are introduced and a separation of collisional and collective correlations is affected. In Section III the plasma wave damping and the dispersion relation are computed in the long wavelength and infinite ion mass limits and a discussion of the results is given in Section IV.

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\* According to reference (6), the result proportional to  $k^2$  published earlier by these authors<sup>8</sup> is not correct.

## II. Mathematical Formulation

We consider a gas of electrons and ions interacting only through Coulomb forces. The ions are assumed to be uniformly distributed and immovable with respect to the high frequency electron oscillations and to be in thermal equilibrium with the electrons in the absence of electron oscillations. The BBGKY hierarchy used to describe such a system can be truncated by an expansion scheme,<sup>9,10</sup> although the expansion parameter is not unique. To first order in the plasma parameter  $g = (\lambda_D^3 n)^{-1}$ , the inverse number of particles in a Debye sphere, the first two members of the hierarchy can be written in linearized form as

$$\begin{aligned} & \left( \frac{\partial}{\partial t} + \vec{v}_1 \cdot \frac{\partial}{\partial \vec{r}_1} \right) f(\vec{r}_1, \vec{v}_1, t) - \frac{ne^2}{m} \frac{\partial f^0}{\partial \vec{v}_1} \cdot \int d\vec{r}_2 d\vec{v}_2 \frac{\partial \phi_{12}}{\partial \vec{r}_1} f(\vec{r}_2, \vec{v}_2, t) \\ & = - \frac{Ze^2 N}{m} \int d\vec{r}_2 d\vec{v}_2 \frac{\partial \phi_{12}}{\partial \vec{r}_1} \frac{\partial Q_{12}}{\partial \vec{v}_1} + \frac{ne^2}{m} \int d\vec{r}_2 d\vec{v}_2 \frac{\partial \phi_{12}}{\partial \vec{r}_1} \cdot \frac{\partial g_{12}}{\partial \vec{v}_1} \end{aligned} \quad (1)$$

$$\begin{aligned} & \left( \frac{\partial}{\partial t} + \vec{v}_1 \cdot \frac{\partial}{\partial \vec{r}_1} + \vec{V}_2 \cdot \frac{\partial}{\partial \vec{R}_2} \right) G_{12}(\vec{r}_1, \vec{R}_2, \vec{v}_1, \vec{V}_2, t) \\ & - \frac{ne^2}{m} \frac{\partial Q_{12}^0}{\partial \vec{v}_1} \int d\vec{r}_3 d\vec{v}_3 \frac{\partial \phi_{13}}{\partial \vec{r}_1} f(\vec{r}_3, \vec{v}_3, t) \\ & + \frac{Zne^2}{M} \frac{\partial Q_{12}^0}{\partial \vec{V}_2} \int d\vec{r}_3 d\vec{v}_3 \frac{\partial \phi_{23}}{\partial \vec{R}_2} f(\vec{r}_3, \vec{v}_3, t) \\ & = - \frac{Ze^2}{M} \frac{\partial \phi_{12}}{\partial \vec{R}_2} \cdot \frac{\partial F^0(V_2)}{\partial \vec{V}_2} f(\vec{r}_1, \vec{v}_1, t) + \frac{ZkT}{\partial \vec{r}_1} \frac{\partial \chi_{12}}{\partial \vec{r}_1} \cdot \frac{\partial f(\vec{r}_1, \vec{v}_1, t)}{\partial \vec{v}_1} F^0(V_2) \\ & + \frac{ne^2}{m} \frac{\partial f^0(v_1)}{\partial \vec{v}_1} \int d\vec{r}_3 d\vec{v}_3 \frac{\partial \phi_{13}}{\partial \vec{r}_1} Q_{23} - \frac{Zne^2}{M} \frac{\partial F^0(v_2)}{\partial \vec{V}_2} \cdot \int d\vec{r}_3 d\vec{v}_3 \frac{\partial \phi_{23}}{\partial \vec{R}_2} Q_{13} \\ & + \frac{Z^2 e^2 N}{M} \frac{\partial F^2(V_2)}{\partial \vec{V}_2} \int d\vec{R}_3 d\vec{V}_3 \frac{\partial \phi_{23}}{\partial \vec{R}_2} Q_{13} \end{aligned} \quad (2)$$

$$\begin{aligned}
& \left( \frac{\partial}{\partial t} + \vec{v}_1 \cdot \frac{\partial}{\partial \vec{r}_1} + \vec{v}_2 \cdot \frac{\partial}{\partial \vec{r}_2} \right) g_{12}(\vec{r}_1, \vec{r}_2, \vec{v}_1, \vec{v}_2, t) \\
& - \frac{ne^2}{m} \frac{\partial g_{12}^0}{\partial \vec{v}_1} \int d\vec{r}_3 d\vec{v}_3 \frac{\partial \phi_{13}}{\partial \vec{r}_1} f(\vec{r}_3, \vec{v}_3, t) \\
& - \frac{ne^2}{m} \frac{\partial g_{12}^0}{\partial \vec{v}_2} \int d\vec{r}_3 d\vec{v}_3 \frac{\partial \phi_{23}}{\partial \vec{r}_2} f(\vec{r}_3, \vec{v}_3, t) \\
& = \frac{e^2}{m_e} \frac{\partial \phi_{12}}{\partial \vec{r}_1} \left[ f(\vec{r}_2, \vec{v}_2, t) \frac{\partial f^0(v_2)}{\partial \vec{v}_1} - f(\vec{r}_1, \vec{v}_1, t) \frac{\partial f^0(v_2)}{\partial \vec{v}_2} \right] \\
& - \frac{\kappa T}{m} \frac{\partial \chi_{12}}{\partial \vec{r}_1} \left[ f^0(v_2) \frac{\partial f(\vec{r}_1, \vec{v}_1, t)}{\partial \vec{v}_1} - f^0(v_1) \frac{\partial f(\vec{r}_2, \vec{v}_2, t)}{\partial \vec{v}_2} \right] \\
& + \frac{ne^2}{m} \frac{\partial f^0(v_1)}{\partial \vec{v}_1} \int d\vec{r}_3 d\vec{v}_3 \frac{\partial \phi_{13}}{\partial \vec{r}_1} g_{23} \\
& + \frac{ne^2}{m} \frac{\partial f^0(v_2)}{\partial \vec{v}_2} \int d\vec{r}_3 d\vec{v}_3 \frac{\partial \phi_{23}}{\partial \vec{r}_2} g_{13} \\
& - \frac{ZNe^2}{m} \frac{\partial f^0(v_1)}{\partial \vec{v}_1} \int d\vec{R}_3 d\vec{V}_3 \frac{\partial \phi_{13}}{\partial \vec{r}_1} Q_{23} \\
& - \frac{ZNe^2}{m} \frac{\partial f^0(v_2)}{\partial \vec{v}_2} \int d\vec{R}_3 d\vec{V}_3 \frac{\partial \phi_{23}}{\partial \vec{r}_2} Q_{23} \tag{3}
\end{aligned}$$

where  $f$ ,  $g$ , and  $Q$  are the perturbed one-electron distribution function, electron-electron and electron-ion correlation functions, respectively. The arguments  $\vec{r}$ ,  $\vec{R}$ ,  $\vec{v}$ ,  $\vec{V}$ ,  $m$ ,  $M$ ,  $n$  and  $N$  refer respectively to the electron and ion position, velocity, mass and number density. The charge  $e$  is the absolute value and  $Z$  is the atomic number of the ions.\* Other

\* We consider one kind of ion only. To extend our discussion to a plasma with many kinds of ions is straightforward.

quantities appearing in equations (1), (2) and (3) are given as follows:

$$f^0(v_1) = \left( \frac{m}{2\pi kT} \right)^{3/2} e^{-\frac{mv_1^2}{2kT}} \quad (4a)$$

$$F^0(V_1) = \left( \frac{M}{2\pi k'T} \right)^{3/2} e^{-\frac{MV_1^2}{2k'T}} \quad (4b)$$

$$g_{12}^0 = \chi_{12} f^0(v_1) f^0(v_2) \quad (4c)$$

$$Q_{12}^0 = -Z\chi_{12} f^0(v_1) F^0(V_2) \quad (4d)$$

$$\chi_{12} = -\frac{e^2}{kT} \frac{1}{|\vec{r}_1 - \vec{r}_2|} e^{-k_D |\vec{r}_1 - \vec{r}_2|} \quad (4e)$$

$$\phi_{12} = \frac{1}{|\vec{r}_1 - \vec{r}_2|} \quad (4f)$$

$$\lambda_D = \frac{1}{k_D} = \left[ \frac{kT}{4\pi e^2 (n + ZN)} \right]^{1/2} \quad (4g)$$

The quantities  $f^0$ ,  $F^0$ ,  $g_{12}^0$  and  $Q_{12}^0$  are, respectively, the equilibrium values for the distribution functions and the electron-electron and electron-ion correlation functions.

The exact solution of the singular integral equations (2) and (3) can be obtained formally as discussed in references (11) and (12). However, because of the complexity of the solution, it is difficult to obtain numerical results and to obtain physical insight into the results. Thus, we will give an approximate treatment to the integral terms on the right-hand side of equations (2) and (3) which will make the analysis more tractable. Noting that, in equilibrium,

$$g^0(1, 2) = \chi_{12}(|\vec{r}_1 - \vec{r}_2|) f^0(v_1) f^0(v_2)$$

we write  $g_{12}$  in the form

$$g_{12} = \chi_{12}(|\vec{r}_1 - \vec{r}_2|) \left[ f(\vec{r}_1, \vec{v}_1, t) f^0(v_2) + f^0(v_1) f(\vec{r}_2, \vec{v}_2, t) \right] + \chi'_{12}(\vec{r}_1, \vec{r}_2, \vec{v}_1, \vec{v}_2, t) \quad (5)$$

and, for the integral terms in equation (2), assume that the last term in equation (5) is negligible. Similarly, for the integral terms in equation (3) we use the approximate solution\*

$$Q_{12} = -Z \chi_{12} f(\vec{r}_1, \vec{v}_1, t) F^0(v_2) \quad (6)$$

With these approximations, and using the relation

$$-\frac{\kappa T}{m} \frac{\partial \chi_{12}}{\partial \vec{r}_1} = \frac{e^2}{m} \frac{\partial \phi_{12}}{\partial \vec{r}_1} + \frac{e^2 n}{m} \left[ \int d\vec{r}_3 \frac{\partial \phi_{13}}{\partial \vec{r}_1} \chi_{23} + Z \int d\vec{r}_3 \frac{\partial \phi_{13}}{\partial \vec{r}_1} \chi_{23} \right] \quad (7)$$

equations (2) and (3) reduce to

$$\begin{aligned} & \left( \frac{\partial}{\partial t} + \vec{v}_1 \cdot \frac{\partial}{\partial \vec{v}_1} + \vec{v}_2 \cdot \frac{\partial}{\partial \vec{v}_2} \right) g_{12} = \\ & = -\frac{\kappa T}{m} \frac{\partial \chi_{12}}{\partial \vec{r}_1} \cdot \left( \frac{\partial}{\partial \vec{v}_1} - \frac{\partial}{\partial \vec{v}_2} \right) \left[ f(\vec{r}_1, \vec{v}_1, t) f^0(v_2) + f^0(v_1) f(\vec{r}_2, \vec{v}_2, t) \right] \\ & + \frac{ne^2}{m} \frac{\partial g_{12}^0}{\partial \vec{v}_1} \int d\vec{r}_3 d\vec{v}_3 \frac{\partial \phi_{13}}{\partial \vec{r}_1} f(\vec{r}_3, \vec{v}_3, t) \\ & + \frac{ne^2}{m} \frac{\partial g_{12}^0}{\partial \vec{v}_2} \int d\vec{r}_3 d\vec{v}_3 \frac{\partial \phi_{23}}{\partial \vec{r}_2} f(\vec{r}_3, \vec{v}_3, t) \end{aligned} \quad (\text{continued on page 7})$$

\* An approximate solution similar to this was used by Kadomtsev<sup>13</sup>. His equations, however, were not linearized.

$$\begin{aligned}
 & + \frac{ne^2}{m} \frac{\partial f^0(v_2)}{\partial \vec{v}_1} f^0(v_2) \int d\vec{r}_3 d\vec{v}_3 \frac{\partial \phi_{13}}{\partial \vec{r}_1} \chi_{23} f(\vec{r}_3, \vec{v}_3, t) \\
 & + \frac{ne^2}{m} \frac{\partial f^0(v_2)}{\partial \vec{v}_2} f^0(v_1) \int d\vec{v}_3 d\vec{r}_3 \frac{\partial \phi_{23}}{\partial \vec{r}_2} \chi_{13} f(\vec{r}_3, \vec{v}_3, t)
 \end{aligned} \quad (8)$$

$$\begin{aligned}
 & \left( \frac{\partial}{\partial t} + \vec{v}_1 \cdot \frac{\partial}{\partial \vec{r}_1} + \vec{v}_2 \cdot \frac{\partial}{\partial \vec{R}_2} \right) Q_{12}(\vec{r}_1, \vec{R}_2; \vec{v}_1, \vec{V}_2; t) \\
 & - \frac{e^2 n}{m} \frac{\partial Q_{12}^0}{\partial \vec{v}_1} \cdot \int d\vec{v}_3 d\vec{r}_3 \frac{\partial \phi_{13}}{\partial \vec{r}_1} f(\vec{r}_3, \vec{v}_3, t) \\
 & + \frac{Ze^2 n}{M} \frac{\partial Q_{12}^0}{\partial \vec{V}_2} \int d\vec{v}_3 d\vec{r}_3 \frac{\partial \phi_{23}}{\partial \vec{R}_2} f(\vec{r}_3, \vec{v}_3, t) \\
 & = \frac{ZkT}{m} \frac{\partial \chi_{12}}{\partial \vec{r}_1} \left( \frac{\partial}{\partial \vec{v}_1} - \frac{m}{M} \frac{\partial}{\partial \vec{V}_2} \right) f(\vec{r}_1, \vec{v}_1, t) F^0(V_2) \\
 & - \frac{Zne^2}{m} \frac{\partial f^0(v_1)}{\partial \vec{v}_1} F^0(V_2) \int d\vec{r}_3 d\vec{v}_3 \frac{\partial \phi_{13}}{\partial \vec{r}_1} \chi_{23} f(\vec{r}_3, \vec{v}_3, t) \\
 & - \frac{Ze^2 n}{M} \frac{\partial F^0(V_2)}{\partial \vec{V}_2} f^0(v_1) \int d\vec{r}_3 d\vec{v}_3 \frac{\partial \phi_{23}}{\partial \vec{R}_2} \chi_{13} f(\vec{r}_3, \vec{v}_3, t) ,
 \end{aligned} \quad (9)$$

The first term on the right-hand side of these equations represents direct two-particle interactions. These terms are responsible, in the homogeneous case,<sup>10</sup> the main structure of the Fokker-Planck type collision integral. They will be referred to henceforth as the collisional correlation terms. On the other hand, the remainder of the terms on the right-hand side of equations (8) and (9) represent integrals of two-particle interactions via a third particle. They will be referred to as collective correlation terms. Writing  $g = g^a + g^b$  and  $Q = Q^a + Q^b$  where superscripts a and b refer to the solutions of the collisional and collective parts respectively.

$$\begin{aligned}
 \left( \frac{\partial}{\partial t} + \vec{v}_1 \cdot \frac{\partial}{\partial \vec{r}_1} \right) f(\vec{r}_1, \vec{v}_1, t) = & \frac{ne^2}{m} \frac{\partial f^0}{\partial \vec{v}_1} \cdot \int d\vec{r}_2 d\vec{v}_2 \frac{\partial \phi_{12}}{\partial \vec{r}_1} f(\vec{r}_2, \vec{v}_2, t) \\
 & + \frac{ne^2}{m} \int d\vec{r}_2 d\vec{v}_2 \frac{\partial \phi_{12}}{\partial \vec{r}_1} \cdot \frac{\partial g_{12}^a(\vec{r}_1, \vec{r}_2, \vec{v}_1, \vec{v}_2, t)}{\partial \vec{v}_1} \\
 & - \frac{Ze^2 N}{m} \int d\vec{R}_2 d\vec{V}_2 \frac{\partial \phi_{12}}{\partial \vec{r}_1} \frac{\partial Q_{12}^a(\vec{r}_1, \vec{R}_2, \vec{v}_1, \vec{V}_2, t)}{\partial \vec{v}_1} \\
 & + \frac{ne^2}{m} \int d\vec{r}_2 d\vec{v}_2 \frac{\partial \phi_{12}}{\partial \vec{r}_1} \cdot \frac{\partial g_{12}^b}{\partial \vec{v}_1} - \frac{ZNe^2}{m} \int d\vec{R}_2 d\vec{V}_2 \frac{\partial \phi_{12}}{\partial \vec{r}_1} \frac{\partial Q_{12}^b}{\partial \vec{v}_1}. \quad (10)
 \end{aligned}$$

Following the ideas of Bogoliubov,<sup>14</sup> we assume that the relaxation time of the collisional correction function is short compared to the relaxation time of the one-electron distribution function. In fact, we will assume it is shorter than the period of a plasma oscillation so that, on the relaxation time scale for  $g^a(t)$  and  $Q^a(t)$ ,  $f(t)$  can be assumed stationary. After a short time the effect of the initial correlation is forgotten and, on the relaxation time scale of  $f$  we will only need the asymptotic value for  $g^a$  and  $Q^a$ , which we call  $g_{\infty}^a$  and  $Q_{\infty}^a$ . Explicitly, equation (10) will be replaced by

$$\begin{aligned}
 \left( \frac{\partial}{\partial t} + \vec{v}_1 \cdot \frac{\partial}{\partial \vec{r}_1} \right) f(\vec{r}_1, \vec{v}_1, t) = & \frac{ne^2}{m} \frac{\partial f^0}{\partial \vec{v}_1} \int d\vec{r}_2 d\vec{v}_2 \frac{\partial \phi_{12}}{\partial \vec{r}_1} f(\vec{r}_2, \vec{v}_2, t) \\
 & + \frac{ne^2}{m} \frac{\partial \phi_{12}}{\partial \vec{r}_1} \frac{\partial}{\partial \vec{v}_1} \int d\vec{v}_2 g_{\infty}^a(\vec{r}_1, \vec{r}_2, \vec{v}_1, \vec{v}_2, t) \\
 & - \frac{Ze^2 N}{m} \int d\vec{R}_2 \frac{\partial \phi_{12}}{\partial \vec{r}_1} \frac{\partial}{\partial \vec{v}_1} \int d\vec{V}_2 Q_{\infty}^a(\vec{r}_1, \vec{R}_2, \vec{v}_1, \vec{V}_2, t) \\
 & + \frac{ne^2}{m} \int d\vec{r}_2 d\vec{v}_2 \frac{\partial \phi_{12}}{\partial \vec{r}_1} \frac{\partial g_{12}^b}{\partial \vec{v}_1} - \frac{ZNe^2}{m} \int d\vec{R}_2 d\vec{V}_2 \frac{\partial \phi_{12}}{\partial \vec{r}_1} \frac{\partial Q_{12}^b}{\partial \vec{v}_1} \quad (11)
 \end{aligned}$$

where, by  $g_{\infty}$  we mean (1) for the calculation of  $g^a(t)$  and  $Q^a(t)$  we will assume  $f$  is stationary in time; and (2) we will then take the limit  $t \rightarrow \infty$ .

In the study of plasma oscillations in the long wavelength limit, it is reasonable to assume that the inhomogeneities in the density occur with a characteristic length many times larger than the Debye distance. However, the collisional correlation terms represent direct interactions which occur almost entirely within a Debye distance. (Note that  $\chi_{12}$  contains a shielding factor.) Thus, we will assume that  $g^a(\vec{r}_1, \vec{r}_2, \dots) = g^a(|\vec{r}_1 - \vec{r}_2|, \dots)$  and  $Q^a(\vec{r}_1, \vec{r}_2, \dots) = Q^a(|\vec{r}_1 - \vec{r}_2|, \dots)$  and that, for the study of these quantities,  $f(\vec{r}_1, \dots)$  can be assumed spatially uniform.

We now introduce the Fourier-Laplace transforms as follows:

$$f(\vec{k}_1, \vec{v}_1, \omega) = \int_{-\infty}^{\infty} d\vec{r}_1 \int_0^{\infty} dt e^{i\vec{k}_1 \cdot \vec{r}_1 + i\omega t} f(\vec{r}_1, \vec{v}_1, t) \quad (12)$$

$$g(\vec{k}_1, \vec{k}_2, \vec{v}_1, \vec{v}_2, \omega) = \int_{-\infty}^{\infty} d\vec{r}_1 d\vec{r}_2 \int_0^{\infty} dt e^{i\vec{k}_1 \cdot \vec{r}_1 + i\vec{k}_2 \cdot \vec{r}_2 + i\omega t} g(\vec{r}_1, \vec{r}_2, \vec{v}_1, \vec{v}_2, t) \quad (13)$$

$$Q(\vec{k}_1, \vec{k}_2, \vec{v}_1, \vec{v}_2, \omega) = \int_{-\infty}^{\infty} d\vec{r}_1 d\vec{r}_2 \int_0^{\infty} dt e^{i\vec{k}_1 \cdot \vec{r}_1 + i\vec{k}_2 \cdot \vec{r}_2 + i\omega t} Q(\vec{r}_1, \vec{r}_2, \vec{v}_1, \vec{v}_2, t) \quad (14)$$

Employing the inverse transforms, equation (11) can be rewritten as

(see next page)

$$\begin{aligned}
 \left( \frac{\partial}{\partial t} + \vec{v}_1 \cdot \frac{\partial}{\partial \vec{r}_1} \right) f(\vec{r}_1, \vec{v}_1, t) = & -\frac{i\omega_p^2}{4\pi} \left( \frac{1}{2\pi} \right)^3 \int d\omega \int d\vec{k}_1 e^{-i\vec{k}_1 \cdot \vec{r}_1 - i\omega t} \frac{\partial}{\partial \vec{v}_1} \left\{ f^0(\vec{v}_1) \rho(\vec{k}_1, \omega) \vec{k}_1 \phi(\vec{k}_1) \right. \\
 & + \left. \left( \frac{1}{2\pi} \right)^3 \int d\vec{k}_2 \vec{k}_2 \phi(\vec{k}_2) \left[ \int d\vec{v}_2 g^b(\vec{k}_1 - \vec{k}_2, \vec{v}_1, \vec{v}_2, \omega) - \int d\vec{v}_2 Q^b(\vec{k}_1 - \vec{k}_2, \vec{v}_1, \vec{v}_2, \omega) \right] \right\} \\
 & - \frac{i\omega_p^2}{4\pi} \left( \frac{1}{2\pi} \right)^3 \frac{\partial}{\partial \vec{v}_1} \int d\vec{k}_1 \vec{k}_1 \phi(\vec{k}_1) \lim_{i\omega \rightarrow 0} (-i\omega) \left[ \int d\vec{v}_2 g^a(-\vec{k}_2, \vec{v}_1, \vec{v}_2, \omega) \right. \\
 & \left. - \int d\vec{v}_2 Q^a(-\vec{k}_2, \vec{v}_1, \vec{v}_2, \omega) \right] \quad (15)
 \end{aligned}$$

where, for the collisional correction terms, we have utilized the fact that

$$\lim_{i\omega \rightarrow 0} (-i\omega) g^a(\vec{k}_1, \vec{v}_1, \omega) = g_{\infty}^a(\vec{k}_1, \vec{v}_1, t) \quad (16)$$

Taking the Fourier-Laplace transforms of equations (8) and (9), we obtain

$$\begin{aligned}
 g^b(\vec{k}_1 - \vec{k}_2, \vec{k}_2, \vec{v}_1, \vec{v}_2, \omega) = & \frac{1}{[\omega + (\vec{k}_1 - \vec{k}_2) \cdot \vec{v}_1 + \vec{k}_2 \cdot \vec{v}_2]} \omega_p^2 \rho(\vec{k}_1, \omega) \left\{ \frac{\vec{k}_1}{k_1^2} \chi(\vec{k}_2) f^0(\vec{v}_2) \frac{\partial f^0(\vec{v}_1)}{\partial \vec{v}_1} \right. \\
 & + \left. \chi(|\vec{k}_1 - \vec{k}_2|) f^0(\vec{v}_1) \frac{\partial f^0(\vec{v}_2)}{\partial \vec{v}_2} \right] \\
 & + \left[ \frac{\vec{k}_1 - \vec{k}_2}{(\vec{k}_1 - \vec{k}_2)^2} \chi(\vec{k}_2) f^0(\vec{v}_2) \frac{\partial f^0(\vec{v}_1)}{\partial \vec{v}_1} + \frac{\vec{k}_2}{k_2^2} \chi(|\vec{k}_1 - \vec{k}_2|) f^0(\vec{v}_1) \frac{\partial f^0(\vec{v}_2)}{\partial \vec{v}_2} \right] \Big\} \\
 & + \frac{1}{[\omega + (\vec{k}_1 - \vec{k}_2) \cdot \vec{v}_1 + \vec{k}_2 \cdot \vec{v}_2]} g_o^b(\vec{k}_1 - \vec{k}_2, \vec{k}_2, \vec{v}_1, \vec{v}_2) \quad (17)
 \end{aligned}$$

$$\begin{aligned}
 Q^b(\vec{k}_1 - \vec{k}_2, \vec{v}_1, \vec{V}_2, \omega) = & -Z \frac{1}{[\omega + (\vec{k}_1 - \vec{k}_2) \cdot \vec{v}_1 + \vec{k}_2 \cdot \vec{V}_2]} \omega_p^2(\vec{k}_1, \omega) \left\{ \frac{\vec{k}_1}{k_1^2} \left[ \chi(k_2) F^0(v_2) \frac{\partial f^0(v_1)}{\partial \vec{v}_1} \right. \right. \\
 & - \frac{Zm}{M} \chi(|\vec{k}_1 - \vec{k}_2|) f^0(v_1) \frac{\partial F^0(V_2)}{\partial \vec{V}_2} \\
 & - \left. \left[ \frac{\vec{k}_1 - \vec{k}_2}{(\vec{k}_1 - \vec{k}_2)^2} \chi(k_2) F^0(V_2) \frac{\partial f^0(v_1)}{\partial \vec{v}_1} + \frac{m}{M} \frac{\vec{k}_2}{k_2^2} \chi(|\vec{k}_1 - \vec{k}_2|) f^0(v_1) \frac{\partial F^0(V_2)}{\partial \vec{V}_2} \right] \right\} \\
 & + \frac{1}{[\omega + (\vec{k}_1 - \vec{k}_2) \cdot \vec{v}_1 + \vec{k}_2 \cdot \vec{V}_2]} Q_o^b(\vec{k}_1 - \vec{k}_2, \vec{k}_2, \vec{v}_1, \vec{v}_2) \quad (18)
 \end{aligned}$$

$$\begin{aligned}
 g^a(-\vec{k}_1, \vec{v}_1, \vec{v}_2, \omega) = & \frac{1}{[\omega - \vec{k}_1 \cdot (\vec{v}_1 - \vec{v}_2)]} \left( \frac{\kappa T}{m} \right) \left( \frac{1}{-i\omega} \right) \vec{k}_1 \chi(k_1) \left( \frac{\partial}{\partial \vec{v}_1} - \frac{\partial}{\partial \vec{v}_2} \right) \\
 & [f(\vec{r}_1, \vec{v}_1, t) f^0(v_2) + f(\vec{r}_1, \vec{v}_2, t) f^0(v_1)] + \frac{i}{[\omega - \vec{k}_1 \cdot (\vec{v}_1 - \vec{v}_2)]} g_o^a(-\vec{k}_1, \vec{v}_1 - \vec{v}_2) \quad (19)
 \end{aligned}$$

$$\begin{aligned}
 Q^a(-\vec{k}_1, \vec{v}_1, \vec{v}_2, \omega) = & \frac{1}{[\omega - \vec{k}_1 \cdot (\vec{v}_1 - \vec{v}_2)]} \left( \frac{\kappa T}{m} \right) \left( \frac{1}{-i\omega} \right) \vec{k}_1 \chi(k_1) \\
 & \times \left( \frac{\partial}{\partial \vec{v}_1} - \frac{m}{M} \frac{\partial}{\partial \vec{V}_2} \right) f(\vec{r}_1, \vec{v}_1, t) F^0(V_2) + \frac{i}{[\omega - \vec{k}_1 \cdot (\vec{v}_1 - \vec{V}_2)]} Q_o^a(-\vec{k}_1, \vec{v}_1, \vec{V}_2) \quad (20)
 \end{aligned}$$

where  $g_o$  and  $Q_o$  are the Fourier transforms of the initial conditions, and where

$$\begin{aligned}
 \omega_p &= \left( \frac{4\pi n e^2}{m} \right)^{1/2} \\
 \chi(k) &= - \frac{k_D^2}{2n(k^2 + k_D^2)} \quad (21)
 \end{aligned}$$

In equations (19) and (20) it is to be noted that, in accordance with earlier remarks, the explicit time and space dependence of the perturbed one-

electron distribution function has been retained. Also, because of the spatial homogeneity,  $f(\vec{r}_2, \vec{v}_2, t)$  has been replaced by  $f(\vec{r}_1, \vec{v}_2, t)$ .

Finally, substituting equations (17)-(20) into equation (15), we obtain

$$\begin{aligned}
 \left( \frac{\partial}{\partial t} + \vec{v}_1 \cdot \frac{\partial}{\partial \vec{r}_1} \right) f(\vec{r}_1, \vec{v}_1, t) = & - \frac{i\omega_p^2}{4\pi} \left( \frac{1}{2\pi} \right)^3 \int d\omega \int d\vec{k}_1 e^{-i\vec{k}_1 \cdot \vec{r}_1 - i\omega t} \rho(\vec{k}_1, \omega) \frac{\partial}{\partial \vec{v}_1} \left\{ f^0(v_1) \vec{k}_1 \phi(k_1) \right. \\
 & + \left( \frac{1}{2\pi} \right)^3 \omega_p^2 \int d\vec{k}_2 \int d\vec{v}_2 \frac{1}{[\omega + (\vec{k}_1 - \vec{k}_2) \cdot \vec{v}_1 + \vec{k}_2 \cdot \vec{v}_2]} (\Psi_{ee} + \Psi_{ei}) \\
 & + \frac{\omega_p^2}{\omega 4\pi} \left( \frac{1}{2\pi} \right)^3 \frac{\partial}{\partial \vec{v}_1} \int d\vec{k}_1 d\vec{k}_2 e^{-i\vec{k}_1 \cdot \vec{r}} \int d\vec{v}_2 \frac{1}{[\omega + (\vec{k}_1 - \vec{k}_2) \cdot \vec{v}_1 + \vec{k}_2 \cdot \vec{v}_2]} g_o^b \\
 & + \left. \int d\vec{v}_2 \frac{1}{[\omega + (\vec{k}_1 - \vec{k}_2) \cdot \vec{v}_1 + \vec{k}_1 \cdot \vec{v}_2]} q_o^b \right\} \\
 & - i \left( \frac{\kappa T}{m} \right) \frac{\omega_p^2}{4\pi} \left( \frac{1}{2\pi} \right)^3 \frac{\partial}{\partial \vec{v}_1} \cdot \left\{ \int d\vec{k}_1 k_1 \phi(k_1) \chi(k_1) \vec{k}_1 \cdot \right. \\
 & \times \int d\vec{v}_2 \delta[\vec{k}_1 \cdot (\vec{v}_1 - \vec{v}_2)] \left( \frac{\partial}{\partial \vec{v}_1} - \frac{\partial}{\partial \vec{v}_2} \right) \left[ f(\vec{r}_1, \vec{v}_1, t) f^0(v_2) + f(\vec{r}_1, \vec{v}_2, t) f^0(v_1) \right] \\
 & + \left. Z \int d\vec{v}_2 \delta[\vec{k}_1 \cdot (\vec{v}_1 - \vec{v}_2)] \left( \frac{\partial}{\partial \vec{v}_1} - \frac{m}{M} \frac{\partial}{\partial \vec{v}_2} \right) f(\vec{r}_1, \vec{v}_1, t) F^0(v_2) \right\} \quad (22)
 \end{aligned}$$

where  $\Psi_{ee}$  and  $\Psi_{ei}$  are those terms contained in  $\{ \}$  bracket in equations (17) and (18), respectively. It is to be noted that the principal value integral associated with the operation  $\lim_{\gamma \rightarrow 0} \frac{1}{\vec{k}_2 \cdot (\vec{v}_1 - \vec{v}_2) - i\gamma} = i\pi \delta[\vec{k}_2 \cdot (\vec{v}_1 - \vec{v}_2)] + \frac{P}{\vec{k}_2 \cdot (\vec{v}_1 - \vec{v}_2)}$  vanishes because the integrand of the  $\vec{k}_2$  integral is antisymmetric.

Finally, by taking the Fourier-Laplace transformation of equation (22), a straightforward calculation leads to the result

$$\rho(\vec{k}_1, \omega) [1 - I - I_{1e} - I_{1i} - I_{2e} - I_{2i} - I_{3e} - I_{3i}] = C(\vec{k}_1) \quad (23)$$

where

$$I = \frac{\omega_p^2}{k_1^2} \int_c d\vec{v}_1 \frac{1}{[\omega + \vec{k}_1 \cdot \vec{v}_1]} \vec{k}_1 \cdot \frac{\partial f^0(v_1)}{\partial \vec{v}_1} \quad (24a)$$

$$I_{1e} = \frac{-i\pi\omega_p^2}{\rho(\vec{k}_1, \omega)} \left( \frac{\kappa T}{m} \right) \left( \frac{1}{2\pi} \right)^3 \int \frac{d\vec{k}_2}{k_2^2} \chi(k_2) \vec{k}_2 \cdot \int_c d\vec{v}_1 \frac{1}{[\omega + \vec{k}_1 \cdot \vec{v}_1]} \frac{\partial}{\partial \vec{v}_1} \\ \times \int d\vec{v}_2 \delta[\vec{k}_2 \cdot (\vec{v}_1 - \vec{v}_2)] \vec{k}_2 \cdot \left( \frac{\partial}{\partial \vec{v}_1} - \frac{\partial}{\partial \vec{v}_2} \right) [f(\vec{k}_1, \vec{v}_1, \omega) f^0(v_2) + f(\vec{k}_1, \vec{v}_2, \omega) f^0(v_1)] \quad (24b)$$

$$I_{1i} = -\frac{Zi\pi\omega_p^2}{\rho(\vec{k}_1, \omega)} \left( \frac{\kappa T}{m} \right) \left( \frac{1}{2\pi} \right)^3 \int \frac{d\vec{k}_2}{k_2^2} \chi(k_2) \vec{k}_2 \cdot \int_c d\vec{v}_1 \frac{1}{[\omega + \vec{k}_1 \cdot \vec{v}_1]} \frac{\partial}{\partial \vec{v}_1} \\ \times \int d\vec{v}_2 \delta[\vec{k}_2 \cdot (\vec{v}_1 - \vec{v}_2)] \vec{k}_2 \cdot \left( \frac{\partial}{\partial \vec{v}_1} - \frac{m}{M} \frac{\partial}{\partial \vec{v}_2} \right) f(\vec{k}_1, \vec{v}_1, \omega) f^0(v_2) \quad (24c)$$

$$I_{2e} = \left( \frac{1}{2\pi} \right)^3 \frac{\omega_p^4}{k_1^2} \int d\vec{k}_2 \frac{\vec{k}_2}{k_2^2} \int_c d\vec{v}_1 \frac{1}{[\omega + \vec{k}_1 \cdot \vec{v}_1]} \frac{\partial}{\partial \vec{v}_1} \\ \times \int d\vec{v}_2 \frac{1}{[\omega + (\vec{k}_1 - \vec{k}_2) \cdot \vec{v}_1 + \vec{k}_2 \cdot \vec{v}_2]} \vec{k}_1 \cdot \left[ \chi(k_2) f^0(v_2) \frac{\partial f^0(v_1)}{\partial \vec{v}_1} \right. \\ \left. + \chi(|\vec{k}_1 - \vec{k}_2|) f^0(v_1) \frac{\partial f^0(v_2)}{\partial \vec{v}_2} \right] \quad (24d)$$

$$I_{2i} = Z \left( \frac{1}{2\pi} \right)^3 \frac{\omega_p^4}{k_1^2} \int d\vec{k}_2 \frac{\vec{k}_2}{k_2^2} \int_c d\vec{v}_1 \frac{1}{(\omega + \vec{k}_1 \cdot \vec{v}_1)} \frac{\partial}{\partial \vec{v}_1} \\ \times \int d\vec{v}_2 \frac{1}{[\omega + (\vec{k}_1 - \vec{k}_2) \cdot \vec{v}_1 + \vec{k}_2 \cdot \vec{v}_2]} \vec{k}_1 \cdot \left[ \chi(k_2) F^0(v_2) \frac{\partial f^0(v_1)}{\partial \vec{v}_1} \right. \\ \left. - \frac{Zm}{M} \chi(|\vec{k}_1 - \vec{k}_2|) f^0(v_1) \frac{\partial F^0(v_2)}{\partial \vec{v}_2} \right] \quad (24e)$$

$$\begin{aligned}
 I_{3e} = & \left(\frac{1}{2\pi}\right)^3 \omega_p^4 \int d\vec{k}_2 \frac{\vec{k}_2}{k_2^2} \int_c d\vec{v}_1 \frac{1}{[\omega + \vec{k}_1 \cdot \vec{v}_1]} \frac{\partial}{\partial \vec{v}_1} \\
 & \times \int d\vec{v}_2 \frac{1}{[\omega + (\vec{k}_1 - \vec{k}_2) \cdot \vec{v}_1 + \vec{k}_2 \cdot \vec{v}_2]} \left[ \frac{\vec{k}_1 - \vec{k}_2}{(\vec{k}_1 - \vec{k}_2)^2} \cdot \chi(k_2) f^o(v_2) \frac{\partial f^o(v_1)}{\partial \vec{v}_1} \right. \\
 & \left. + \frac{\vec{k}_2}{k_2^2} \chi(|\vec{k}_1 - \vec{k}_2|) f^o(v_1) \frac{\partial f^o(v_2)}{\partial \vec{v}_2} \right] \quad (24f)
 \end{aligned}$$

$$\begin{aligned}
 I_{3i} = & Z \left(\frac{1}{2\pi}\right)^3 \omega_p^4 \int d\vec{k}_2 \frac{\vec{k}_2}{k_2^2} \int_c d\vec{v}_1 \frac{1}{[\omega + \vec{k}_1 \cdot \vec{v}_1]} \frac{\partial}{\partial \vec{v}_1} \\
 & \times \int d\vec{V}_2 \frac{1}{[\omega + (\vec{k}_1 - \vec{k}_2) \cdot \vec{v}_1 + \vec{k}_2 \cdot \vec{V}_2]} \frac{\vec{k}_1 - \vec{k}_2}{(\vec{k}_1 - \vec{k}_2)^2} \chi(k_2) F^o(V_2) \frac{\partial f^o(v_1)}{\partial \vec{v}_1} \\
 & + \frac{m}{M} \frac{\vec{k}_2}{k_2^2} \chi(|\vec{k}_1 - \vec{k}_2|) f^o(v_1) \frac{\partial F^o(V_2)}{\partial \vec{V}_2} \quad (24g)
 \end{aligned}$$

$$\begin{aligned}
 C(\vec{k}_1) = & i \int_c d\vec{v}_1 \frac{f_o(\vec{k}_1, \vec{v}_1)}{\omega + \vec{k}_1 \cdot \vec{v}_1} + i \left(\frac{1}{2\pi}\right)^3 \omega_p^2 \int_c d\vec{v}_1 \frac{1}{\omega + \vec{k}_1 \cdot \vec{v}_1} \int d\vec{k}_2 \frac{\vec{k}_2}{k_2^2} \cdot \frac{\partial}{\partial \vec{v}_1} \\
 & \times \left[ \int d\vec{v}_2 \frac{g_o^b(\vec{k}_1 - \vec{k}_2, \vec{v}_1, \vec{v}_2, \omega)}{\omega + (\vec{k}_1 - \vec{k}_2) \cdot \vec{v}_1 + \vec{k}_2 \cdot \vec{v}_2} - \int d\vec{V}_2 \frac{G^b(\vec{k}_1 - \vec{k}_2, \vec{v}_1, \vec{V}_2, \omega)}{\omega + (\vec{k}_1 - \vec{k}_2) \cdot \vec{v}_1 + \vec{k}_2 \cdot \vec{V}_2} \right] \quad (24h)
 \end{aligned}$$

The path of integration for the  $\vec{v}_1$  integrals has been discussed by Landau.<sup>15</sup> We have found that, for the computation of  $I_{se}$  and  $I_{si}$  ( $s = 1, 2, 3$ ), in the long wavelength limit, the indented part of the Landau contour only gives small, high order contributions. Thus, we will simplify our discussion of these integrals by replacing the Landau contour, which is in the complex  $u = (\vec{k}_1 \cdot \vec{v}_1)/k$  plane, by the real axis.

In Section III, the relationship of equation (23) to the damping and the dispersion relation for electron plasma oscillations will be developed.

Then the collisional terms  $I_{1e}$  and  $I_{1i}$  will be studied and it will be shown that within certain approximations, these quantities are independent of  $\rho(\vec{k}_1, \omega)$ . All of the  $I$  terms will be evaluated and, finally, numerical results for the damping and the dispersion relation will be given.

### III. Numerical Calculations

It is seen by examination of equation (23) that the dispersion equation is given by

$$1 - I - I_{1e} - I_{1i} - I_{2e} - I_{2i} - I_{3e} - I_{3i} = 0 \quad (25)$$

Following the method presented by Jackson<sup>16</sup>, we expand

$$I\left(\frac{\omega_r}{k_1} + i\frac{\gamma}{k_1}\right) \approx \frac{\omega_p^2}{k_1^2} \left\{ P \int du \frac{\partial \bar{f}^0(u)}{\partial u} \frac{1}{\left(u + \frac{\omega_r}{k_1}\right)} - i\pi \frac{\partial \bar{f}^0(u)}{\partial u} \right|_{u = -\frac{\omega_r}{k_1}} - \frac{i\gamma}{k_1} P \int du \frac{\partial \bar{f}^0(u)}{\partial u} \frac{1}{\left(u + \frac{\omega_r}{k_1}\right)^2} \right\} \quad (26)$$

where we have defined  $\omega = \omega_r + i\gamma$  and assumed a priori that  $\frac{\gamma}{\omega_r} \ll 1$

as  $k_1 \rightarrow 0$ . The quantity  $\bar{f}^0(u)$  is defined by  $\bar{f}^0(u) = \int_{-\infty}^{\infty} d^2 v_{\perp} f^0(v_{\perp})$

where  $\vec{v}_{\perp}$  is the two-dimensional vector component of  $\vec{v}_1$  normal to  $\vec{k}_1$ .

Substituting equation (26) into (25) and setting real and imaginary parts equal to zero, we obtain

$$1 - \frac{\omega_p^2}{k_1^2} P \int du \frac{\partial \bar{f}^0(u)}{\partial u} \frac{1}{\left(u + \frac{\omega_r}{k_1}\right)} - \text{Re}(I_{1e} + I_{1i} + I_{2e} + I_{2i} + I_{3e} + I_{3i}) = 0 \quad (27a)$$

$$\gamma = \left[ -\frac{\omega_p^2}{k_1^2} \pi \frac{\partial \bar{f}^0(u)}{\partial u} \right]_{u=-\frac{\omega_r}{k_1}} + \mathcal{I}m(I_{1e} + I_{1i} + I_{2e} + I_{2i} + I_{3e} + I_{3i}) \left[ \frac{\omega_p^2}{k_1^3} P \int du \frac{\partial \bar{f}^0(u)}{\partial u} \frac{1}{\left(u + \frac{\omega_r}{k_1}\right)^2} \right]^{-1} \quad (27b)$$

Using the dispersion relation, equation (27a), it can be shown that

$$\frac{\omega_p^2}{k_1^3} P \int du \frac{\partial \bar{f}^0(u)}{\partial u} \frac{1}{\left(u + \frac{\omega_r}{k_1}\right)^2} = \frac{2}{\omega_r} \left(1 - \frac{k_1}{\omega_r} \frac{d\omega_r}{dk_1}\right)^{-1}$$

so that, in the long wavelength limit,

$$\omega_r^2 = \omega_p^2 \left[ 1 + \frac{k_1^2}{k_D^2} + \frac{\omega_r^2}{\omega_p^2} \mathcal{R}e(I_{1e} + I_{1i} + I_{2e} + I_{2i} + I_{3e} + I_{3i}) \right] \quad (28)$$

$$\gamma = -\frac{\omega_r}{2k_1^2} \left(1 - \frac{k_1}{\omega_r} \frac{d\omega_r}{dk_1}\right) \omega_p^2 \pi \frac{\partial \bar{f}^0(u)}{\partial u} \Big|_{u=-\frac{\omega_r}{k_1}} + \frac{\omega_r}{2} \left(1 - \frac{k_1}{\omega_r} \frac{d\omega_r}{dk_1}\right) \mathcal{I}m(I_{1e} + I_{1i} + I_{2e} + I_{2i} + I_{3e} + I_{3i}) \quad (29)$$

$$\text{where } k_D^2 = \frac{2m\omega_p^2}{KT}$$

#### a) Contribution from the Collisional Part

According to the definitions given in Section II,

$$\begin{aligned} I_1 = I_{1e} + I_{1i} &= \frac{i\pi\omega_p^2}{2\rho(k_1, \omega)} \left(\frac{KT}{m}\right) \frac{k_D^2}{n} \left(\frac{1}{2\pi}\right)^3 \int d\vec{v}_1 \frac{1}{(\omega + \vec{k}_1 \cdot \vec{v}_1)^2} \int \frac{d\vec{k}_2}{k_2^2} \frac{\vec{k}_1 \cdot \vec{k}_2}{k_2^2 + k_D^2} \\ &\times \left\{ \int d\vec{v}_2 \delta[\vec{k}_2 \cdot (\vec{v}_1 - \vec{v}_2)] \vec{k}_2 \cdot \left(\frac{\partial}{\partial \vec{v}_1} - \frac{\partial}{\partial \vec{v}_2}\right) [f(\vec{k}_1, \vec{v}_1, \omega) f^0(v_2) \right. \\ &\quad \left. + f(\vec{k}_1, \vec{v}_2, \omega) f^0(v_1)] \right. \\ &\quad \left. + Z \int d\vec{V}_2 \delta[\vec{k}_2 \cdot (\vec{v}_1 - \vec{V}_2)] \vec{k}_2 \cdot \left(\frac{\partial}{\partial \vec{v}_1} - \frac{m}{M} \frac{\partial}{\partial \vec{V}_2}\right) f(\vec{k}_1, \vec{v}_1, \omega) F^0(V_2) \right\} \quad (30) \end{aligned}$$

where an integration by parts on  $\vec{v}_1$  has been performed and the value of  $\chi(k_2)$  has been used. Define  $\vec{g} = \vec{v}_1 - \vec{v}_2$  and  $\vec{G} = \vec{v}_1 - \vec{V}_2$ . Write  $\vec{k}_2 = k_2 \hat{k}_2 = k_2(\hat{k}_{2\parallel} + \hat{k}_{2\perp})$ , where  $\hat{k}_2$  is a unit vector in the direction of  $\vec{k}_2$ ,  $\hat{k}_{2\parallel}$  is the component of  $\hat{k}_2$  parallel to  $\vec{g}$  (or  $\vec{G}$ ) and  $\hat{k}_{2\perp}$  is the component perpendicular to  $\vec{g}$  (or  $\vec{G}$ ). Writing  $\vec{k}_2$  in cylindrical coordinates and performing the  $k_{2\parallel}$  integration we find

$$I_1 = \frac{i(\frac{1}{2\pi})^2}{2\rho(\vec{k}_1, \omega)} \frac{\omega_p^4}{\omega_n^2} k_1 \int d\vec{v}_1 \frac{1}{(1 + \frac{\vec{k}_1 \cdot \vec{v}_1}{\omega})^2} \int_0^{k_{\max}} dk_{2\perp} \frac{k_{2\perp}}{k_{2\perp}^2 + k_D^2} \int d\phi$$

$$\times \left\{ \int d\vec{v}_2 \frac{1}{g} \hat{k}_1 \cdot \hat{k}_{2\perp} \hat{k}_{2\perp} \cdot \left( \frac{\partial}{\partial \vec{v}_1} - \frac{\partial}{\partial \vec{v}_2} \right) \left[ f(\vec{k}_1, \vec{v}_1, \omega) f^0(\vec{v}_2) + f(\vec{k}_1, \vec{v}_2, \omega) f^0(\vec{v}_1) \right] \right.$$

$$\left. + Z \int d\vec{V}_2 \frac{1}{G} \hat{k}_1 \cdot \hat{k}_{2\perp} \hat{k}_{2\perp} \cdot \left( \frac{\partial}{\partial \vec{v}_1} - \frac{m}{M} \frac{\partial}{\partial \vec{V}_2} \right) f(\vec{k}_1, \vec{v}_1, \omega) F^0(\vec{V}_2) \right\} \quad (31)$$

where, to prevent a close collision divergence in the  $k$  integration, we have cut off the integral. For electron-electron encounters the usual cut-off is taken to be the Landau distance  $k_L^{-1} \equiv \left( \frac{KT}{e} \right)^{-1}$ , the average distance of closest approach for a binary collision. For electron-ion collisions, however, where the formation of bound states is a possibility, the cut-off is not well defined. One possible cut-off suggested in references (4) and (6) is the de Broglie wavelength  $k_T^{-1} = \left( \frac{\hbar^2}{2mKT} \right)^{1/2}$ . Presumably, if the particles do not approach closer than this distance, and  $k_T^{-1} \gg r_0$ , the range of the nuclear forces, a bound state will not be formed.

Performing the  $k_{2\perp}$  integration, and using the fact that

$$\frac{1}{g} \int d\phi \hat{k}_{2\perp} \hat{k}_{2\perp} = \pi \frac{g^2 \hat{I} - \vec{g} \vec{g}}{g^3}$$

where  $\underline{I}$  is the unit dyadic, we obtain

$$\begin{aligned}
 I_1 = & \frac{i}{\rho(\vec{k}_1, \omega)} \left( \frac{1}{8\pi} \right) \frac{\omega_p^4}{\omega_n^2} \ln \left( \frac{k_M}{k_D} \right) \int d\vec{v}_1 \frac{1}{\left( 1 + \frac{\vec{k}_1 \cdot \vec{v}_1}{\omega} \right)^2} \\
 & \times \left\{ \int d\vec{v}_2 \vec{k}_1 \cdot \frac{g^2 \underline{I} - \vec{g} \vec{g}}{g^3} \cdot \left( \frac{\partial}{\partial \vec{v}_1} - \frac{\partial}{\partial \vec{v}_2} \right) [f(\vec{k}_1, \vec{v}_1, \omega) f^0(v_2) + f(\vec{k}_1, \vec{v}_1, \omega) f^0(v_1)] \right. \\
 & \left. + Z \int d\vec{V}_2 \vec{k}_1 \cdot \frac{G^2 \underline{I} - \vec{G} \vec{G}}{G^3} \cdot \left( \frac{\partial}{\partial \vec{v}_1} - \frac{m}{M} \frac{\partial}{\partial \vec{V}_2} \right) f(\vec{k}_1, \vec{v}_1, \omega) F^0(V_2) \right\} \quad (32)
 \end{aligned}$$

where  $k_M = k_L$  for electron-electron collisions and  $k_M = k_T$  for electron-ion encounters. Utilizing the relation

$$\frac{1}{g^3} [g^2(\vec{a} \cdot \vec{b}) - (\vec{g} \cdot \vec{a})(\vec{g} \cdot \vec{b})] = \frac{1}{\pi^2} \int d^3 \ell e^{-i\vec{\ell} \cdot (\vec{v}_1 - \vec{v}_2)} \frac{(\vec{\ell} \cdot \vec{a})(\vec{\ell} \cdot \vec{b})}{\ell^4} \quad (33)$$

and the one obtained by changing  $g \rightarrow G$ , equation (32) can be written

$$\begin{aligned}
 I_1 = & \frac{i\pi^2}{\rho(\vec{k}_1, \omega)} \left( \frac{1}{2\pi} \right)^3 \frac{\omega_p^4}{\omega_n^2} \ln \left( \frac{k_M}{k_D} \right) \int du \frac{1}{\left( 1 + \frac{k_1 u}{\omega} \right)^2} \int d^2 v_{1\perp} \\
 & \times \int \frac{d^3 \ell}{\ell^4} (\vec{k}_1 \cdot \vec{\ell}) \vec{\ell} \cdot \left\{ \int d\vec{v}_2 e^{-i\vec{\ell} \cdot (\vec{v}_1 - \vec{v}_2)} \left( \frac{\partial}{\partial \vec{v}_1} - \frac{\partial}{\partial \vec{v}_2} \right) [f(\vec{k}_1, \vec{v}_1, \omega) f^0(v_2) \right. \\
 & \left. + f(\vec{k}_1, \vec{v}_2, \omega) f^0(v_1)] + Z \int d\vec{V}_2 e^{-i\vec{\ell} \cdot (\vec{v}_1 - \vec{V}_2)} \delta(V_2) \frac{\partial}{\partial \vec{v}_1} f(\vec{k}_1, \vec{v}_1, \omega) \right\} \quad (34)
 \end{aligned}$$

where for convenience, we have taken the infinite ion mass limit  $\frac{m}{M} \rightarrow 0$ ,  $F^0(V_2) = \delta(V_2)$ . Note that  $\vec{v}_1$  has been broken into two components,  $u$  and  $\vec{v}_{1\perp}$ . The first is parallel to  $k_1$  and the second is normal to it.

To go further we shall approximate  $f$  by its Vlasov equation solution. This is justifiable since we only require our results accurate to first order in  $\epsilon = \frac{1}{\lambda_D^3 n}$ . (cf. ref. 3). Recall that the integration path for the  $u$  integral is along the real axis. This is equivalent to taking the principal value integral for  $u$ , where  $\omega = \omega_r$ . In the long wavelength limit, keeping terms up to order  $k_1^2$ , the  $\vec{v}_2$ ,  $\vec{v}_1$  and  $\vec{I}$  integrations can be performed straightforwardly in that order, with the result

$$I_1 = -\frac{i}{\pi^{3/2}} \frac{\omega_p^3}{\omega_r^3} \frac{\tilde{k}_D^3}{n} \left\{ \frac{Z\sqrt{2}}{12} \ln \left( \frac{1}{\sqrt{2}} \frac{k_T}{\tilde{k}_D} \right) + \frac{\omega_p^2}{\omega_r^2} \frac{k_1^2}{\tilde{k}_D^2} \left[ \frac{Z\sqrt{2}}{3} \ln \left( \frac{1}{\sqrt{2}} \frac{k_T}{\tilde{k}_D} \right) + \frac{2}{15} \ln \left( \frac{1}{\sqrt{2}} \frac{k_L}{\tilde{k}_D} \right) \right] \right\} \quad (35)$$

where

$$\tilde{k}_D^2 = \frac{4\pi n e^2}{kT} = \frac{1}{2} k_D^2$$

Then

$$\begin{aligned} \gamma_a = \frac{\omega_r}{2} \left( 1 - \frac{k_1}{\omega_r} \frac{d\omega_r}{dk_1} \right) \text{Im } I_1 = & -\frac{\omega_p Z}{12\sqrt{2}} \frac{\tilde{k}_D^3}{n} \ln \left( 0.7071 \frac{k_T}{\tilde{k}_D} \right) \\ & + \frac{1}{\pi^{3/2}} \frac{\omega_p Z}{6\sqrt{2}} \frac{k_1^2}{\tilde{k}_D^2} \frac{\tilde{k}_D^3}{n} \ln \left( 0.7071 \frac{k_T}{\tilde{k}_D} \right) \\ & - \frac{\omega_p}{15} \frac{1}{\pi^{3/2}} \frac{k_1^2}{\tilde{k}_D^2} \frac{\tilde{k}_D^3}{n} \ln \left( 0.7071 \frac{k_L}{\tilde{k}_D} \right) \end{aligned} \quad (36)$$

The positive contribution arises from the factor  $(1 - \frac{k_1}{\omega_r} \frac{d\omega_r}{dk_1})$  and from the expansion of the factor  $\frac{\omega_p^3}{\omega_r^3}$  which appears in front of the wavelength independent term.

(b) Contribution from the Collective Part

From Section II,  $I_{2e}$  and  $I_{2i}$  can be rewritten

$$I_{2e} = - \left( \frac{1}{2\pi} \right)^3 \frac{\omega_p^4}{k_1^2} \int d\vec{k}_2 \frac{\vec{k}_1 \cdot \vec{k}_2}{k_2^2} \int d\vec{v}_1 \frac{1}{(\omega + \vec{k}_1 \cdot \vec{v}_1)^2} \int_0^\infty dt e^{i[\omega + (\vec{k}_1 - \vec{k}_2) \cdot \vec{v}_1] t} \\ \times \int d\vec{v}_2 e^{i\vec{k}_2 \cdot \vec{v}_2 t} \left[ i\chi(k_2) f^o(v_2) \vec{k}_1 \cdot \frac{\partial f^o(v_1)}{\partial \vec{v}_1} + i\chi(|\vec{k}_1 - \vec{k}_2|) f^o(v_1) \frac{\partial f^o(v_2)}{\partial \vec{v}_2} \right] \quad (37)$$

$$I_{2i} = -Z \left( \frac{1}{2\pi} \right)^3 \frac{\omega_p^4}{k_1^2} \int d\vec{k}_2 \frac{\vec{k}_1 \cdot \vec{k}_2}{k_2^2} \int d\vec{v}_1 \frac{1}{(\omega + \vec{k}_1 \cdot \vec{v}_1)^2} \int_0^\infty dt e^{i[\omega + (\vec{k}_1 - \vec{k}_2) \cdot \vec{v}_1] t} \\ \times \int d\vec{v}_2 e^{i\vec{k}_2 \cdot \vec{v}_2 t} \left[ i\chi(k_2) F^o(v_2) \vec{k}_1 \cdot \frac{\partial f^o(v_1)}{\partial \vec{v}_1} - i \frac{Zm}{M} \chi(|\vec{k}_1 - \vec{k}_2|) f^o(v_1) \frac{\partial F^o(v_2)}{\partial \vec{v}_2} \right] \quad (38)$$

Letting  $\vec{v}_1 = u\hat{k}_1 + \vec{v}_{1\perp}$  where  $\hat{k}_1$  is a unit vector in the direction of  $\vec{k}_1$  and  $\vec{v}_{1\perp}$  is the component of  $\vec{v}_1$  perpendicular to  $\vec{k}_1$ , it is a straightforward calculation to perform the  $\vec{v}_2$  (and  $\vec{V}_2$ ) and  $\vec{v}_{1\perp}$  integrations. In the infinite ion mass limit we obtain

$$I_{2e} = - \left( \frac{1}{2\pi} \right)^2 \frac{\omega_p^4}{\omega_r^2} \int dt dk_2 d\xi k_2 \xi e^{-\frac{k_2^2 t^2 \beta}{2} (2 - \xi^2)} \\ \times P \int du f^o(u) \frac{e^{i[\omega + (k_1 - k_2 \xi)u] t}}{(1 + \frac{k_1 u}{\omega})^2} \left[ (k_1 - k_2 \xi) t \chi(k_2) \right. \\ \left. + \frac{2ik_1 \chi(k_2)}{\omega_r (1 + \frac{k_1 u}{\omega_r})} + k_2 \xi t \chi(|\vec{k}_1 - \vec{k}_2|) \right] \quad (39)$$

$$I_{2i} = -Z \left( \frac{1}{2\pi} \right)^2 \frac{\omega_p^4}{\omega_r} \int dt \int dk_2 d\xi k_2 \xi e^{-\frac{k_2^2 t^2 \beta (1-\xi^2)}{2}} \chi(k_2) \\ \times P \int du f^0(u) \frac{e^{i[\omega + (k_1 - k_2 \xi)u]t}}{\left(1 + \frac{k_1 u}{\omega_r}\right)^2} \left[ (k_1 - k_2 \xi)t + \frac{2ik_1}{\omega_r \left(1 + \frac{k_1 u}{\omega_r}\right)} \right] \quad (40)$$

where  $\beta = \frac{\kappa T}{m}$ ,  $\xi = \frac{\vec{k}_1 \cdot \vec{k}_2}{k_1 k_2}$  and where, in accordance with earlier remarks, we are taking the principal value integral for the  $u$  integration.

We now perform a long wavelength expansion, noting that

$$e^{i[\omega + (k_1 - k_2 \xi)u]t} \simeq \left[ 1 + ik_1 t u - \frac{k_1^2 t^2 u^2}{2} \right] e^{i[\omega - k_2 \xi u]t}$$

and

$$\chi(\vec{k}_1 - \vec{k}_2) \simeq \chi(k_2) \left[ 1 + \frac{2k_1 k_2 \xi - k_1^2}{k_D^2 + k_2^2} \right]$$

After performing the  $u$  and  $\xi$  integrations, in that order, we find

$$I_{2e} = -\frac{2i}{(2\pi)^2} \frac{\omega_p^2}{\omega^2} \frac{k_1^2}{k_D^2} \frac{k_D^3}{n} \int d\tau e^{i\tilde{\omega}\tau} \int dy \frac{y^2}{1+y^2} e^{-2y^2\tau^2} \left[ \frac{i}{6(1+y^2)} \right. \\ \left. + \frac{i}{\tilde{\omega}^2} + \frac{2}{3} \frac{\tau}{\tilde{\omega}} + \frac{4}{5} \frac{\tau}{\tilde{\omega}} \frac{y^2}{1+y^2} - \frac{2i}{5} \tau^2 \frac{y^2}{1+y^2} - \frac{i}{3} \tau^2 \right] \quad (41)$$

$$I_{2i} = \frac{Z}{3\pi^2} \frac{\omega_p^2}{\omega^2} \frac{k_D^3}{n} \int d\tau \tau e^{i\tilde{\omega}\tau} \int dy \frac{y^2}{1+y^2} e^{-y^2\tau^2} \\ - \frac{Z}{2\pi^2} \frac{\omega_p^2}{\omega^2} \frac{k_1^2}{k_D^2} \frac{k_D^3}{n} \int_0^\infty d\tau \tau e^{i\tilde{\omega}\tau} \int_0^\infty dy \frac{y^2}{1+y^2} e^{-y^2\tau^2} \\ \times \left[ \frac{3}{\tilde{\omega}^2} - \frac{2i\tau}{\tilde{\omega}} - \frac{6}{5} \frac{y^2\tau^2}{\tilde{\omega}^2} + \frac{4iy^2\tau^3}{5\tilde{\omega}} + \frac{y^2\tau^4}{5} \right] \quad (42)$$

where  $\tilde{\omega} = \frac{\omega}{\omega_p}$ ,  $\tau = \omega_p t$ , and  $y = k_2/k_D$ . It is seen that a term independent of  $k_1$  survives for the ion part whereas, because of cancellation, this term does not appear in the electron part.

All of these integrals can be reduced to two types, viz.

$$M_j = \int_0^\infty d\tau e^{i\tilde{\omega}\tau} \tau^j = i^{j+1} \frac{j!}{\tilde{\omega}^{j+1}}$$

and

$$N_j = \int_0^\infty d\tau e^{i\Omega\tau} \tau^j \int_0^\infty dy \frac{e^{-y^2\tau^2}}{1+y^2} \quad (43)$$

where  $\Omega = \frac{\tilde{\omega}}{\sqrt{2}}$  for the integrals in  $I_{2e}$  and  $\Omega = \tilde{\omega}$  for the integrals in  $I_{2i}$ . The integrals  $N_j$  can be performed analytically. The method for doing this is given briefly in Appendix A. It will be treated in greater detail in a future publication.

After considerable labor, we obtain the results

$$I_{2e} = -\frac{\omega_p^2}{\omega_r^2} \frac{k_1^2}{\tilde{k}_D^2} \frac{\tilde{k}_D^3}{n} \frac{1}{\pi^{3/2}} [0.48 + i0.97] \quad (44)$$

$$I_{2i} = -\frac{\omega_p^2}{\omega_r^2} \frac{Z\tilde{k}_D^3}{n} \frac{1}{\pi^{3/2}} [0.0475 + i\frac{\omega_p}{\omega_r} 0.0308] - Z \frac{\omega_p^2}{\omega^2} \frac{k_1^2}{\tilde{k}_D^2} \frac{\tilde{k}_D^3}{n} \frac{1}{\pi^{3/2}} [-0.2120 - i0.1681], \quad (45)$$

The reduction of the terms  $I_{3e}$  and  $I_{3i}$  follows exactly the same procedure just outlined for  $I_{2e}$  and  $I_{2i}$ . Hence we only give the results:

$$I_{3e} = -\frac{\omega_p^2}{\omega_r^2} \frac{k_1^2}{\tilde{k}_D^2} \frac{\tilde{k}_D^3}{n} \frac{1}{\pi^{3/2}} [-0.2012 + i0.0321] \quad (46)$$

$$I_{3i} = -Z \frac{\omega_p^2}{\omega^2} \frac{k_1^2}{\tilde{k}_D^2} \frac{\tilde{k}_D^3}{n} \frac{1}{\pi^{3/2}} [0.0000 + i0.1743] \quad (47)$$

contribution independent of  $k$  is found.

Collecting all of these terms we now obtain

$$\begin{aligned} \gamma_b &= \frac{\omega_r}{2} \left( 1 - 3 \frac{k_1^2}{\tilde{k}_D^2} \right) \text{Im} (I_{2e} + I_{2i} + I_{3e} + I_{3i}) \\ &= -0.031 \frac{Z\omega_p}{\pi^{3/2}} \frac{\tilde{k}_D^3}{n} - \frac{\omega_p}{\pi^{3/2}} \frac{k_1^2}{\tilde{k}_D^2} \frac{\tilde{k}_D^3}{n} [1.00 + 0.158 Z] \end{aligned} \quad (48)$$

$$\frac{\omega^2}{\omega_p^2} = \left\{ 1 - \frac{Z}{\pi^{3/2}} 0.0475 \frac{\tilde{k}_D^3}{n} + 3 \frac{k_1^2}{\tilde{k}_D^2} \left[ 1 - (0.0929 + 0.0734Z) \frac{\tilde{k}_D^3}{n} \right] \right\}, \quad (49)$$

Part of the  $k^2$  contribution to the damping was obtained by expanding a factor  $\omega_p^2/\omega_r^2$  which appeared in the lowest order term of  $I_{2i}$ .

#### IV. Discussion and Conclusions

The effects of electron-ion and electron-electron correlations on the damping and dispersion relation of plasma oscillations have been studied by use of the BBGKY hierarchy. We have seen that the linearized equations for the electron-ion and electron-electron correlation functions, which are integro-differential equations, could be simplified by substituting an approximate solution of these functions into the integral terms. It was then possible to separate the correlation functions into a collisional part and a collective part, each affecting the plasma oscillations in different ways.

In order to facilitate further discussion, we summarize our results as follows:

$$\frac{\gamma}{\omega_p} = \frac{\gamma_\ell}{\omega_p} + \frac{\gamma_a}{\omega_p} + \frac{\gamma_b}{\omega_p} = \frac{\gamma_\ell}{\omega_p} - \frac{Z}{\pi^{3/2} 12\sqrt{2}} \frac{\tilde{k}_D^3}{n} \ln \left( 0.92 \frac{k_T}{\tilde{k}_D} \right) + \frac{k_1^2}{\tilde{k}_D^2} \frac{\tilde{k}_D^3}{n} \left[ \frac{1}{6\sqrt{2} \pi^{3/2}} Z \ln \left( 0.363 \frac{k_T}{\tilde{k}_D} \right) - \frac{1}{15 \pi^{3/2}} \ln \left( 1320 \frac{k_L}{\tilde{k}_D} \right) \right] \quad (50)$$

$$\frac{\gamma_\ell}{\omega_p} = - \left( \frac{\pi}{8} \right)^{1/2} \frac{\tilde{k}_D^3}{k_1^3} \exp \left( - \frac{\tilde{k}_D^2}{2k_1^2} \right) \exp \left( - \frac{3}{2} \right) \quad (50a)$$

$$\frac{\gamma_a}{\omega_p} = - \frac{Z}{\pi^{3/2} 12\sqrt{2}} \frac{\tilde{k}_D^3}{n} \ln \left( 0.707 \frac{k_T}{\tilde{k}_D} \right) + \left( \frac{3}{6} - \frac{2}{6} \right) \frac{Z}{\sqrt{2} \pi^{3/2}} \frac{k_1^2}{\tilde{k}_D^2} \frac{\tilde{k}_D^3}{n} \ln \left( 0.707 \frac{k_T}{\tilde{k}_D} \right) - \frac{1}{15 \pi^{3/2}} \frac{k_1^2}{\tilde{k}_D^2} \frac{\tilde{k}_D^3}{n} \ln \left( 0.707 \frac{k_L}{\tilde{k}_D} \right) \quad (50b)$$

$$\frac{\gamma_b}{\omega_p} = - \frac{Z}{\pi^{3/2} 12\sqrt{2}} \frac{\tilde{k}_D^3}{n} \ln (1.30) - \frac{Z}{\pi^{3/2} 6\sqrt{2}} \frac{k_1^2}{\tilde{k}_D^2} \frac{\tilde{k}_D^3}{n} \ln (1.95) - \frac{1}{15 \pi^{3/2}} \frac{k_1^2}{\tilde{k}_D^2} \frac{\tilde{k}_D^3}{n} \ln (1870) \quad (50c)$$

$$\frac{\omega^2}{\omega_p^2} = \left\{ \left( 1 - \frac{Z}{\pi^{3/2}} 0.0475 \frac{\tilde{k}_D^3}{n} \right) + 3 \frac{k_1^2}{\tilde{k}_D^2} \left[ 1 - (0.0929 + 0.0734Z) \frac{\tilde{k}_D^3}{n} \right] \right\} \quad (51)$$

where  $\omega_p = \left( \frac{4\pi n e^2}{m} \right)^{1/2}$ ;  $\tilde{k}_D = \left( \frac{4\pi n e^2}{kT} \right)^{1/2}$ ;  $k_T = \frac{(2mkT)^{1/2}}{\hbar}$ ;  $k_L = \left( \frac{e^2}{kT} \right)^{1/2}$ .

The damping  $\gamma_\ell$  is the usual Landau damping which can be obtained in the "self consistent field" theory;  $\gamma_a$  is the damping resulting from collisional correlation effects, and  $\gamma_b$  is the damping which results from collective correlation effects. In equation (50), the combined effect of these terms is given.

We see that pair correlations affect plasma oscillations in several striking ways. Most important is the appearance of a damping which does not vanish in the limit  $k \rightarrow 0$ , and which will be the principal damping mechanism for  $k \ll k_D$ . The presence of this term emphasizes the importance of electron-ion correlations, even in the infinite ion mass limit, for electron-electron correlation damping is proportional to  $k^2$ . The result which we have found can be compared with the result

$$\frac{\gamma}{\omega_p} = \frac{1}{12\sqrt{2}\pi^{3/2}} \frac{\tilde{k}_D^3}{n} \ln\left(0.74 \frac{k_T}{\tilde{k}_D}\right)$$

obtained from Perel and Eliashberg<sup>4\*</sup>. The agreement is seen to be quite good especially in light of the ambiguity in the upper limit cut-off.

Another interesting effect of electron-ion correlations, as seen from equation (51), is the small correction to the Langmuir plasma frequency which has not been discussed previously. It seems reasonable to interpret this correction as a small increase in the effective mass of the electrons. Also, we observe that the correction which is proportional to  $k^2 \tilde{\lambda}_D^2$  is negative.

We have already mentioned that Ichikawa<sup>1</sup> and Willis<sup>2</sup> studied the effects of electron-electron correlations on plasma waves. The electron-electron correlation equation which they solved is found by dropping all but the self-consistent field terms from equation (2). Thus the terms responsible for dynamical shielding (the integral terms) have been neglected and the terms responsible for the main structure of the Fokker-Planck

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\* Their conductivity is related to our damping according to  $\gamma = 2\pi \text{Re } \sigma(\omega_p)$ .

equation have been dropped. The results which are equivalent to theirs are found from our  $I_{2e}$  calculation. The comparison of the results can be made from the following list:

$$\omega^2 = \omega_p^2 \left[ 1 + 3 \frac{k^2}{k_D^2} \left( 1 + a \sqrt{n \left( \frac{e^2}{kT} \right)^3} \right) \right]$$

$$\gamma = \gamma_\ell + b \omega_p \frac{k^2}{k_D^2} \sqrt{n \left( \frac{e^2}{kT} \right)^3}$$

$a = 1.05$	$b = -0.028$	Ichikawa
$a = 9.2$	$b = -17.6$	Willis
$a = -1.2795$	$b = -3.89$	Present Authors

where  $\gamma_\ell$  is the classical Landau damping which is a negative quantity.

# APPENDIX A

We wish to evaluate the integral

$$N_j = \int_0^\infty d\tau e^{i\Omega\tau} \tau^j \int_0^\infty dy \frac{e^{-\tau^2 y^2}}{1+y^2} \quad (A-1)$$

We write

$$\int_0^\infty dy \frac{e^{-\tau^2 y^2}}{1+y^2} = \frac{\pi}{2} e^{\tau^2} \operatorname{erfc} \tau \quad (A-2)$$

where

$$\operatorname{erfc} \tau = \left[ 1 - \frac{2}{\sqrt{\pi}} \int_0^\tau dx e^{-x^2} \right]$$

Let  $p = -i\Omega$ . Then

$$N_j = \frac{\pi}{2} \mathfrak{L} \left\{ \tau^j e^{\tau^2} \operatorname{erfc} \tau \right\} \quad (A-3)$$

where  $\mathfrak{L}$  is the Laplace transform operation.

From formula (5), page 176, of reference (17), we find

$$\mathfrak{L} \left\{ e^{\tau^2} \operatorname{erfc} \sqrt{\tau} \right\} = \frac{1}{\sqrt{p} (\sqrt{p} + 1)} \quad (A-4)$$

Then, using formula (23), page 131, of reference (17), we obtain

$$N_j = \sqrt{\pi} p^{-j} \left( \frac{1}{2} \right)^{j+1} \int_0^\infty dZ \frac{Z^j}{p+Z} e^{-\frac{Z^2}{4}} H_j \left( \frac{Z}{2} \right) \quad (A-5)$$

where

$$H_j(Z) = (-1)^j e^{Z^2} \frac{d^j}{dZ^j} e^{-Z^2}$$

Although  $p$  is complex, the real part, which gives a damping, leads only to higher order corrections for the damping and dispersion. Thus

we set  $p = -i\Omega$  where  $\Omega$  is now taken as real. Then

$$N_j = \frac{(i)^j \sqrt{\pi}}{2^{j+1}} \frac{1}{\Omega^j} \left[ \int_0^\infty dZ \frac{Z^{j+1}}{Z^2 + \Omega^2} e^{-\frac{Z^2}{4}} H_j\left(\frac{Z}{2}\right) + i\Omega \int_0^\infty dZ \frac{Z^j}{Z^2 + \Omega^2} e^{-\frac{Z^2}{4}} H_j\left(\frac{Z}{2}\right) \right]. \quad (\text{A-6})$$

These integrals have been computed for  $\Omega = 1$  and  $1/\sqrt{2}$  and  $j = 0, 1, 2, 3, 4, 5$  and the results will be given in a future publication.

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